A Generic Exact Solver for Vehicle Routing and Related Problems

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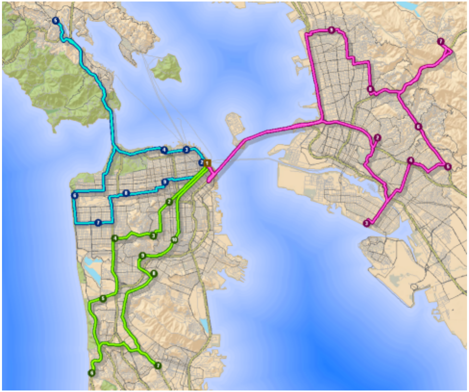
Vehicle Routing Problem (VRP)

One of the most widely investigated optimization problems:

- Google Scholar finds 728 works published in 2017 (2 per day!) containing both “vehicle” and “routing” in the title

Direct application in the real-world systems that distribute goods and provide services



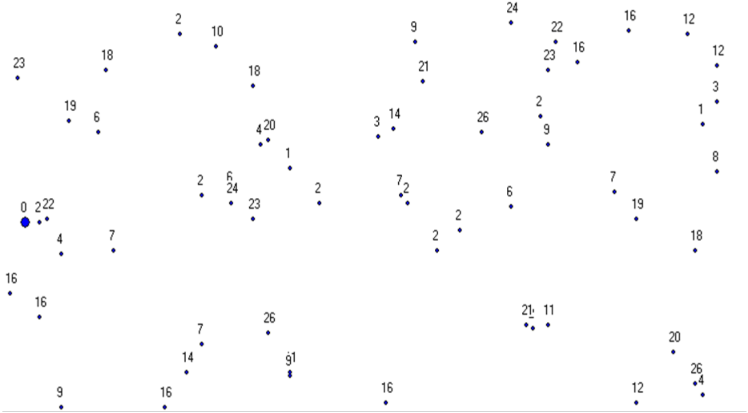
Vehicle Routing Problem (VRP)

VRP literature recognizes hundreds of variants. For example, there are variants that consider:

- Vehicle capacities
- Time windows
- Heterogeneous fleet
- Multiple depots
- Split delivery, pickup and delivery, backhauling
- Optional customer service
- Arc routing (Ex: garbage collection)
- etc, etc

Capacited Vehicle Routing Problem (CVRP)

First (Dantzig and Ramser [1959]) and **most basic variant**: each customer i has a demand d_i, vehicles have a capacity Q



62 customers, indicated demands, $Q = 100$

Capacited Vehicle Routing Problem (CVRP)

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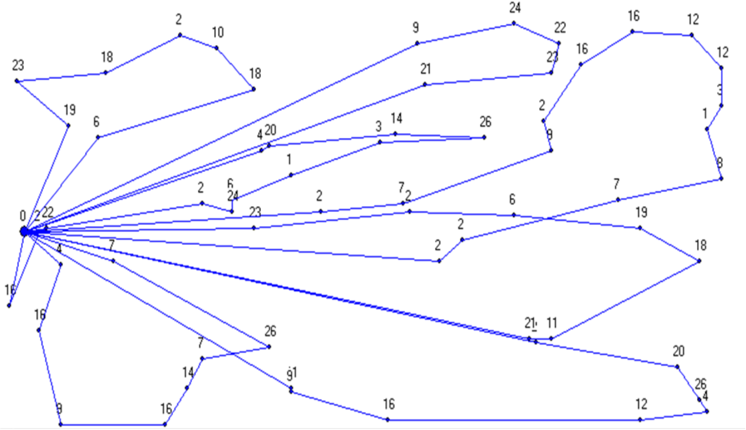
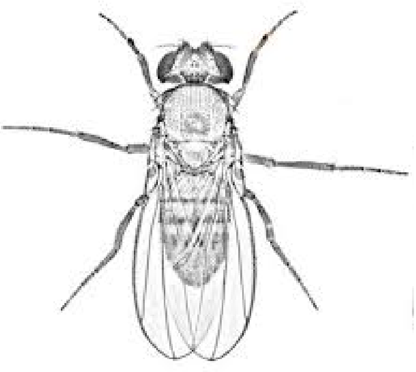


62 customers, indicated demands, $Q = 100$

Why we care so much about CVRP?



Drosophila Melanogaster

Why we care so much about CVRP?

Common strategy in scientific research:

1. Study the simplest (but still representative!) case of a phenomenon
2. Generalize the discoveries for more complex cases

A recent breakthrough on CVRP

A complex Branch-Cut-and-Price (BCP) algorithm increased the size of the largest solved instance from 150 to 360 customers

- Combines and enhances ideas from several previous authors
- The new concept of **limited-memory cut** proved to be crucial

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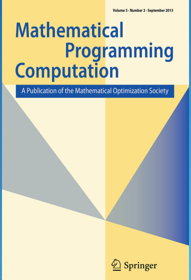
This certificate is awarded at the 23rd International Symposium on Mathematical Programming, Bordeaux, France, July 2018



MPC Best Paper in 2017

The editorial board of MPC has chosen

Improved Branch-Cut-and-Price for Capacitated Vehicle Routing

by **Diego Pecin, Artur Pessoa, Marcus Poggi** and **Eduardo Uchoa**
MPC, volume 9, pp. 61-100, March 2017

Comparable recent advances in other classical variants

VRPTW Diego Pecin, Claudio Contardo, Guy Desaulniers, and Eduardo Uchoa. New enhancements for exactly solving the vehicle routing problem with time windows. *INFORMS Journal on Computing*, 29: 489–502, 2017a

HFVRP Artur Pessoa, Ruslan Sadykov, and Eduardo Uchoa. Enhanced branch-cut-and-price algorithm for heterogeneous fleet vehicle routing problems. *European Journal of Operational Research*, 270: 530–543, 2018a

CARP Diego Pecin and Eduardo Uchoa. Comparative analysis of capacitated arc routing formulations for designing a new branch-cut-and-price algorithm. *Transportation Science*, (Forthcoming), 2019

Good News

For all those variants, it is now possible to solve almost all instances with up to **200 customers**, sometimes taking **hours or even days** of CPU time.

More interestingly, typical instances with **100 customers**, that a few years ago would take hours or days, are now solvable in less than **1 minute**.

It seems that, for the first time, many instances of practical size are within the range of exact algorithms

Bad News

Designing and coding a state-of-the-art BCP algorithm for a particular variant is a very demanding task, measured in several work-months of a skilled team

In effect, this prevents the practical use of those algorithms in real world problems, that actually, seldom correspond exactly to one of the most classical variants

Proposal of this work

A BCP solver for a generic model that encompasses a wide class of VRPs and even some other kinds of problems

Incorporates key elements found in the best recent VRP algorithms,

- ng-path relaxation,
- Rank-1 cuts with limited memory,
- Route enumeration,

generalized through the new concept of **packing set**

Very careful BCP implementation for good performance in such a general setting

The Basic Model

Basic Model: Graphs for Resource Constrained Shortest Path (RCSP) generation

Define a set R of resources, divided into **main resources** R^M and **secondary resources** R^N

Define **directed graphs** $G^k = (V^k, A^k)$, $k \in K$:

- **Special vertices** v^k_{source}, v^k_{sink}
- **Non-negative arc consumption** $q_{a,r} \in \mathbb{R}_+$, $a \in A^k$, $r \in R^M$
  - cycles with zero main resource consumption should not exist
- **Unrestricted arc consumption** $q_{a,r} \in \mathbb{R}$, $a \in A^k$, $r \in R^N$
- **Accumulated resource consumption intervals** $[l_{a,r}, u_{a,r}]$, $a \in A^k$, $r \in R$
  - May also be defined on vertices ($[l_{v,r}, u_{v,r}]$, $v \in V^k$, $r \in R$)

Let $V = \cup_{k \in K} V^k$ and $A = \cup_{k \in K} A^k$

Basic Model: Graphs for Resource Constrained Shortest Path (RCSP) generation

Resource Constrained Path

A path $p = (v^k_{\text{source}} = v_0, a_1, v_1, \ldots, a_{n-1}, v_{n-1}, a_n, v_n = v^k_{\text{sink}})$ over G^k is resource constrained iff for every $r \in R$, the accumulated resource consumption $S_{j,r}$ at visit j, $0 \leq j \leq n$, where $S_{0,r} = 0$ and $S_{j,r} = \max\{l_{a_j,r}, S_{j-1,r} + q_{a_j,r}\}$, does not exceed $u_{a_j,r}$.

- For each $k \in K$, P^k is the set of all resource constrained paths in G^k
- $P = \cup_{k \in K} P^k$

Basic Model: Variables and Mappings

Define continuous and/or integer variables:

1. Mapped x variables
   - Each variable x_j, $1 \leq j \leq n_1$, is mapped into a non-empty set $M(j) \subseteq A$.
   - The inverse mapping of arc a is $M^{-1}(a) = \{j | a \in M(j)\}$.
2. Additional (non-mapped) y variables

Basic Model: Formulation

h_a^p = how many times arc a is used in path p

$$\text{Min} \quad \sum_{j=1}^{n_1} c_j x_j + \sum_{s=1}^{n_2} f_s y_s \tag{1a}$$

$$\text{S.t.} \quad \sum_{j=1}^{n_1} \alpha_{ij} x_j + \sum_{s=1}^{n_2} \beta_{is} y_s \geq d_i, \qquad i = 1, \ldots, m, \tag{1b}$$

$$x_j = \sum_{k \in K} \sum_{p \in P^k} \left(\sum_{a \in M(j)} h_a^p \right) \lambda_p, \quad j = 1 \ldots, n_1, \tag{1c}$$

$$L^k \leq \sum_{p \in P^k} \lambda_p \leq U^k, \qquad k \in K, \tag{1d}$$

$$\lambda_p \in \mathbb{Z}_+, \qquad p \in P, \tag{1e}$$

$$x_j \in \mathbb{N},\ y_s \in \mathbb{N} \qquad j = 1, \ldots, \bar{n}_1,\ s = 1, \ldots, \bar{n}_2 \tag{1f}$$

(1b) may even contain an exponential # of constraints, provided that suitable separation routines are given

Solving the Basic Model: Substituting the x variables and relaxing integrality

$$\text{Min} \quad \sum_{k\in K}\sum_{p\in P^k}\left(\sum_{j=1}^{n_1} c_j \sum_{a\in M(j)} h_a^p\right)\lambda_p + \sum_{s=1}^{n_2} f_s y_s \tag{2a}$$

$$\text{S.t.} \quad \sum_{k\in K}\sum_{p\in P^k}\left(\sum_{j=1}^{n_1} \alpha_{ij} \sum_{a\in M(j)} h_a^p\right)\lambda_p + \sum_{s=1}^{n_2} \beta_{is} y_s \geq d_i, \quad i = 1,\dots,m, \tag{2b}$$

$$L^k \leq \sum_{p\in P^k} \lambda_p \leq U^k, \quad k \in K, \tag{2c}$$

$$\lambda_p \geq 0, \quad p \in P. \tag{2d}$$

Dual variables of (2b): π_i, $1 \leq i \leq m$.
Dual variables of (2c): ν_+^k and ν_-^k, $k \in K$.

Solving the Basic Model: BP or robust BCP

Reduced cost of arc $a \in A$:

$$\bar{c}_a = \sum_{j \in M^{-1}(a)} c_j - \sum_{i=1}^{m} \sum_{j \in M^{-1}(a)} \alpha_{ij} \pi_i .$$

Reduced cost of path $p = (v_0, a_1, v_1, \ldots, a_{n-1}, v_{n-1}, a_n, v_n) \in P^k$:

$$\bar{c}(p) = \sum_{j=1}^{n} \bar{c}_{a_j} - \nu_+^k - \nu_-^k .$$

Pricing subproblems correspond to finding, for each $k \in K$, a path $p \in P^k$ with minimum reduced cost.

The basic model leads to a standard **Branch-and-Price (BP)** algorithm (or to a **robust Branch-Cut-and-Price (BCP)** if some constraints in (1b) are separated)

Including Advanced Elements: Packing Sets

Collection of Packing Sets

Let $\mathcal{B} \subset 2^A$ be a collection of mutually disjoint subsets of A such that the constraints:

$$\sum_{a \in B} \sum_{p \in P} h_a^p \lambda_p \leq 1, \quad B \in \mathcal{B}, \tag{3}$$

are satisfied by at least one optimal solution (x^*, y^*, λ^*) of Formulation (1). Then, $\mathcal{B}$ is a **collection of packing sets**.

The definition of a proper $\mathcal{B}$ is part of the modeling

Generalizing ng-paths

The bounds obtained by linear relaxation (2) can often be improved by only working with elementary paths. However, the resulting pricing problems may be intractable. The ng-paths (Baldacci et al. [2011a]) obtain a good compromise between bound quality and pricing difficulty

Generalized ng-paths

For each arc $a \in A$, let $NG(a) \subseteq \mathcal{B}$ denote the *ng*-set of a. An *ng*-path may use two arcs belonging to the same packing set B, but only if the subpath between those two arcs passes by an arc a such that $B \notin NG(a)$.

Generalizing Limited Memory Rank-1 Cuts

The Rank-1 Cuts (R1Cs) (Pecin et al. [2017c]) are a generalization of the Subset Row Cuts proposed by Jepsen et al. [2008] for variants modeled as set partitioning problems. They are further generalized as follows.

Consider a collection of packing sets $\mathcal{B}$ and non-negative multipliers ρ_B for each $B \in \mathcal{B}$. A Chvátal-Gomory rounding of Constraints (3) yields a generalized R1C:

$$\sum_{p \in P} \left\lfloor \sum_{B \in \mathcal{B}} \rho_B \sum_{a \in B} h_a^p \right\rfloor \lambda_p \leq \left\lfloor \sum_{B \in \mathcal{B}} \rho_B \right\rfloor . \quad (4)$$

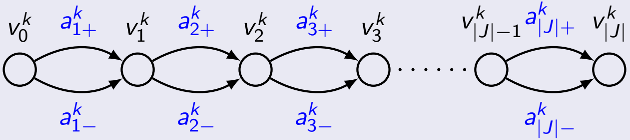
Limited-memory is used to avoid excessive impact in the pricing

Model Examples

Example 1: Generalized Assignment Problem (GAP)

- Set T of tasks; set K of machines; capacity Q^k, $k \in K$; assignment cost c_t^k and machine load w_t^k, $t \in T$, $k \in K$
- Find an assignment of tasks to machines such that the total load in each machine does not exceed its capacity, with minimum total cost

RCSP Graphs G^k (modeling binary knapsack problems)



$V^k = \{v_t^k : t = 0, \ldots, |T|\}$, $A^k = \{a_{t+}^k = (v_{t-1}^k, v_t^k),$
$a_{t-}^k = (v_{t-1}^k, v_t^k) : t = 1, \ldots, |T|\}$, $v_{\text{source}}^k = v_0^k$, $v_{\text{sink}}^k = v_{|T|}^k$.
$R = R^M = \{1\}$; $q_{a_{t+}^k,1} = w_t^k, q_{a_{t-}^k,1} = 0, t \in T$;
$[l_{v_t^k,1}, u_{v_t^k,1}] = [0, Q^k]$, $t \in T \cup \{0\}$.

Example 1: Generalized Assignment Problem (GAP)

Formulation

Binary variables x_t^k, $t \in T$, $k \in K$.

$$\text{Min} \quad \sum_{t \in T} \sum_{k \in K} c_t^k x_t^k \tag{5a}$$

$$\text{S.t.} \quad \sum_{k \in K} x_t^k = 1, \qquad t \in T; \tag{5b}$$

$L^k = 0$, $U^k = 1$, $k \in K$; Mapping $M(x_t^k) = \{a_{t+}^k\}, t \in T, k \in K$

Packing Sets

$\mathcal{B} = \cup_{t \in T}\{\{a_{t+}^k : k \in K\}\}$

Example 2: Bin Packing and Vector Packing

- Set T of items; set D of dimensions; bin capacities Q^d, $d \in D$; item weight w_t^d, $t \in T$, $d \in D$ ($|D| = 1$ is Bin Packing)
- Find a packing using the minimum number of bins, such that, for each dimension, the total weight of the items in a bin does not exceed its capacity.

Single RCSP Graph G (index k omitted)

$V = \{v_t : t = 0, \ldots, |T|\}$,
$A = \{a_{t+} = (v_{t-1}, v_t), a_{t-} = (v_{t-1}, v_t) : t = 1, \ldots, |T|\}$,
$v_{\text{source}} = v_0$, $v_{\text{sink}} = v_{|T|}$.
$R = R^M = D$; $q_{a_{t+},d} = w_t^d, q_{a_{t-},d} = 0, t \in T, d \in D$;
$[l_{v_t,d}, u_{v_t,d}] = [0, Q^d]$, $t \in T \cup \{0\}, d \in D$.

Example 2: Bin Packing and Vector Packing

Formulation

Binary variables x_t, $t \in T \cup \{0\}$.

$$\text{Min} \quad x_0 \tag{6a}$$

$$\text{S.t.} \quad x_t = 1, \qquad t \in T; \tag{6b}$$

$L = 0$, $U = \infty$;

Mapping $M(x_0) = \{a_{1+}, a_{1-}\}$, $M(x_t) = \{a_{t+}\}, t \in T$.

Packing Sets

$\mathcal{B} = \cup_{t \in T}\{\{a_{t+}\}\}$.

Example 3: Pickup and Delivery VRP with Time Windows

- Directed graph $G' = (V', A')$, $V' = \{0\} \cup P' \cup D'$, where $P' = \{1, \ldots, n\}$ are pickups and $D' = \{n+1, \ldots, 2n\}$ are the corresponding deliveries; vehicle capacities Q; traveling cost c_a and time t_a , $a \in A'$; positive demands d_v, $v \in P'$ ($d_v = -d_{v-n}$, $v \in D'$); and time windows $[l'_v, u'_v]$, $v \in V'$
- Find routes performing all pickups and deliveries respecting capacities and time windows, minimizing total costs.

Single RCSP Graph G (index k omitted)

$V = V' \cup \{2n+1\}$, $A = (A' \setminus \{(v, 0) : v \in D'\}) \cup \{(v, 2n+1) : v \in D'\}$, $v_{\text{source}} = v_0$, $v_{\text{sink}} = v_{2n+1}$. $R^M = \{n+2\}$; $R^N = \{1, \ldots, n+1\}$; $q_{(v,v'),v'} = 1$, if $v' \in P'$, $q_{(v,v'),v'-n} = -1$, if $v' \in D'$, and $q_{(v,v'),n+1} = d_{v'}$, $(v, v') \in A$; $q_{a,n+2} = t_a$, $a \in A$; all other resource consumptions are zero; $u_{v,r} = 1$, $r = 1, \ldots, n$, $u_{v,n+1} = u_{2n+1,n+1} = Q$ and $(l_{v,n+2}, u_{v,n+2}) = (l'_v, u'_v)$, $v \in P' \cup D'$; all other resource bounds are zero.

Example 3: Pickup and Delivery VRP with Time Windows

Formulation

Binary variables x_a, $a \in A$.

$$\text{Min} \quad \sum_{a \in A} c_a x_a \tag{7a}$$

$$\text{S.t.} \quad \sum_{a \in \delta^-(v)} x_a = 1, \qquad v \in P'; \tag{7b}$$

$L = 0,\ U = \infty$;

Mapping $M(x_a) = \{a\}$, $a \in A$.

Packing Sets

$\mathcal{B} = \cup_{v \in V} \{\{\delta^-(v)\}\}$.

The example illustrates the use of secondary resources with both positive and negative consumptions as a modeling device

Example 4: Capacitated Arc Routing (CARP)

- Graph $G' = (V', E)$, $V' = \{0, \ldots, n\}$, 0 is the depot vertex; positive cost c_e and non-negative demand d_e, $e \in E$, set of required edges $S = \{e \in E \mid d_e > 0\}$; vehicle capacity Q
- Find a minimum cost set of routes serving the demands in all required edges. Edges in a route can be traversed either serving or deadheading (not serving).

Single RCSP Graph G (index k omitted)

For $i, j \in V'$, let $D(i,j) \subseteq E$ be a cheapest path from i to j, with cost $C(i,j)$. Define a dummy required edge $r_0 = (0, 0')$ and $S_0 = S \cup \{r_0\}$. For each $r = (w_1, w_2) \in S_0$, define $o(r, w_1) = w_2$ and $o(r, w_2) = w_1$. $V = \{v_r^w : r \in S_0, w \in r\}$,
$A = \{(v_{r_1}^{w_1}, v_{r_2}^{z_1}), (v_{r_1}^{w_1}, v_{r_2}^{z_2}), (v_{r_1}^{w_2}, v_{r_2}^{z_1}), (v_{r_1}^{w_2}, v_{r_2}^{z_2}) : r_1 = (w_1, w_2), r_2 = (z_1, z_2) \in S_0\}$, $v_{\text{source}} = v_{r_0}^{0}$, $v_{\text{sink}} = v_{r_0}^{0'}$;
$R = R^M = \{1\}$; for $a = (v_{r_1}^{w}, v_{r_2}^{z}) \in A$, $q_{a,1} = d_{r_2}$;
$l_{v,1} = 0, u_{v,1} = Q, v \in V$.

Example 4: Capacitated Arc Routing (CARP)

Formulation

Binary variables x_a, $a \in A$. For $a = (v_{r_1}^w, v_{r_2}^z) \in A$,
$c_a = C(w, o(r_2, z)) + c_{r_2}$.

$$\text{Min} \quad \sum_{a \in A} c_a x_a \tag{8a}$$

$$\text{S.t.} \quad \sum_{a \in \delta^-(\{v_r^{w_1}, v_r^{w_2}\})} x_a = 1, \qquad r = (w_1, w_2) \in S, \tag{8b}$$

plus Rounded Capacity Cuts (Laporte and Nobert [1983]) and Lifted Odd-Cutsets (Belenguer and Benavent [1998], Bartolini et al. [2013]); $L = 0$, $U = \infty$; Mapping $M(x_a) = \{a\}$, $a \in A$.

Packing Sets

$\mathcal{B} = \cup_{r=(w_1,w_2) \in S} \{\delta^-(\{v_r^{w_1}, v_r^{w_2}\})\}$

In this example, a complex transformation is needed to fit the problem into the proposed model

BCP Implementation and Computational Experiments

BCP Implementation Choices

- Pricing: bucket graph (defined by the main resources) labeling Sadykov et al. [2017] over dynamic *ng*-paths Baldacci et al. [2011a] Roberti and Mingozzi [2014] Bulhoes et al. [2018b]
- Robust Cuts: built-in separation of rounded capacity cuts Laporte and Nobert [1983] Lysgaard et al. [2004]
- Non-Robust Cuts: Limited-Memory Rank-1 Jepsen et al. [2008] Pecin et al. [2014] Pecin et al. [2017c] Pecin et al. [2017a]
- Dual stabilization: Automatic smoothing Wentges [1997] Pessoa et al. [2018b]
- Reduced cost fixing: bucket graph fixing Ibaraki and Nakamura [1994] Irnich et al. [2010] Pessoa et al. [2010] Sadykov et al. [2017]
- Enumeration of elementary routes: Baldacci et al. [2008] Contardo and Martinelli [2014]
- Strong branching: Multi-phase Røpke [2012] Pecin et al. [2014]
- Built-in heuristics: diving heuristic Sadykov et al. [2018]

Computational Experiments

- Solver optimization algorithms coded in C++ over BaPCod package (Vanderbeck et al. [2018])
- IBM CPLEX 12.8 used as LP solver
- Experiments run on Intel Xeon E5-2680 v3 2.50 GHz processors
- The models are defined using either a C++ interface or a Julia–JuMP (Dunning et al. [2017]) based interface.

Tests over 13 problems, including: Multi-Depot VRP (MDVRP), (Capacitated) Team Orienteering Problem (CTOP/TOP), Capacitated Profitable Tour Problem (CPTP), and VRP with Service Level constraints (VRPSL).

- Single parameterization per problem

Computational results

Problem	Data set	#	T.L.	Gen. BCP	Best Published		2nd Best Published	
CVRP	E-M	12	10h	12 (61s)	**12 (49s)**	Pecin et al. [2017b]	10 (432s)	Contardo et al. [2014]
	X	58	60h	**36 (147m)**	34 (209m)	Uchoa et al. [2017]	—	
VRPTW	Sol Hard	14	1h	**14 (5m)**	13 (17m)	Pecin et al. [2017a]	9 (39m)	Baldacci et al. [2011a]
	Hom 200	60	30h	**56 (21m)**	50 (70m)	Pecin et al. [2017a]	7 (-)	Kallehauge et al. [2006]
HFVRP	Golden	40	1h	**40 (144s)**	39 (287s)	Pessoa et al. [2018a]	34 (855s)	Baldacci et al. [2009]
MDVRP	Cordeau	11	1h	**11 (6m)**	11 (7m)	Pessoa et al. [2018a]	9 (25m)	Contardo et al. [2014]
PDPTW	RC	40	1h	**40 (5m)**	33 (17m)	Gschwind et al. [2018]	32 (14m)	Baldacci et al. [2011b]
	LiLim	30	1h	3 (56m)	**23 (20m)**	Baldacci et al. [2011b]	18 (27m)	Gschwind et al. [2018]
TOP	Chao 4	60	1h	**55 (8m)**	39 (15m)	Bianchessi et al. [2018]	30 (-)	El-Hajj et al. [2016]
CTOP	Archetti	14	1h	**13 (7m)**	7 (34m)	Archetti et al. [2013]	6 (35m)	Archetti et al. [2009]
CPTP	Archetti	28	1h	**24 (9m)**	0 (1h)	Bulhoes et al. [2018a]	0 (1h)	Archetti et al. [2013]
VRPSL	Bulhoes	180	2h	**159 (16m)**	49 (90m)	Bulhoes et al. [2018a]	—	
GAP	OR-Lib D	6	2h	5 (40m)	**5 (30m)**	Posta et al. [2012]	5 (46m)	Avella et al. [2010]
	Nauss	30	1h	**25 (23m)**	1 (58m)	Gurobi [2017]	0 (1h)	Nauss [2003]
VPP	1,4,5,9	40	1h	**38 (8m)**	13 (50m)	Heßler et al. [2018]	10 (53m)	Brandão et al. [2016]
BPP	Falk T	80	10m	80 (16s)	**80 (1s)**	Brandão et al. [2016]	80 (24s)	Belov et al. [2006,16]
	Hard28	28	10m	28 (17s)	**28 (7s)**	Belov et al. [2006,16]	26 (14s)	Brandão et al. [2016]
	AI	250	1h	**160 (25m)**	116 (35m)	Belov et al. [2006,16]	100 (40m)	Brandão et al. [2016]
	ANI	250	1h	103 (35m)	**164 (35m)**	Clautiaux et al. [2017]	51 (48m)	Belov et al. [2006,16]
CARP	Eglese	24	30h	**22 (36m)**	22 (43m)	Pecin et al. [2019]	10 (237m)	Bartolini et al. [2013]

Table: Generic solver vs best specific solvers on 13 problems.

Additional Experiments on Some Open CVRP instances

Instance	Prev. BKS	Root LB	Nodes	Total Time	OPT
X-n284-k15	20226	20168	940	11.0 days	**20215**
X-n322-k28	29834	29731	1197	5.6 days	**29834**
X-n393-k38	38260	38194	1331	5.8 days	**38260**
X-n469-k138	221909	221585	8964	15.2 days	**221824**
X-n584-k50	86710	86650	337	2.0 days	**86700**

- Long runs, parameters calibrated per instance

Conclusions and Perspectives

Releasing the VRP Solver over Coluna.jl

We will release the VRP solver for academic use as soon as Coluna.jl is sufficiently performing and reliable:

- VRP specific algorithms bundled in a single pre-compiled library
- BCP handled by Coluna.jl (open source)
- Julia–JuMP user interface for modeling (open source), including several demos

JUMP

1. Modeling a typical VRP variant requires around 100 lines of Julia code (not counting input/output). A user can build a good working solver for a new variant in 1 day
2. Computer experiments and parameter tuning may be needed for an improved performance
3. In some cases, separation routines for problem specific cuts are needed for top performance

Room for Creative Modeling

We believe users may find original ways (transformations) of fitting new problems in the proposed model

- Not only VRP variants, possibly also problems from scheduling, network design, etc.

Since VRP solving technology is quite advanced, there is a chance of obtaining better-than-existing-methods performance

Thank you

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